VISCOSITY OF WATER AT TEMPERATURES OF −20 TO 150°C

A. A. Aleksandrov and M. S. Trakhtengerts

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An equation for dynamic viscosity of water at atmospheric pressure is presented together with tables of dynamic and kinematic viscosity.

Data on water viscosity at atmospheric pressure are widely employed in viscosimetry for calibration of viscosimeters and performance of relative measurements. A list of the basic experimental studies on water viscosity at atmospheric pressure or close thereto and temperatures differing from 20°C is presented in Table 1. With the exception of Leroux's study [10], measurements were performed with some modification of the capillary viscosimeter, using both absolute and relative methods. In the latter case the quantity measured was the relative viscosity

$$\mu^{rel} = \mu/\mu_{20}, \qquad (1)$$

with different values of μ_{20} chosen by different authors. Table 1 also shows the error ascribed to the measurements by the authors, the indicated value being the error in μ^{rel} in the case of relative measurements.

Analysis of the experimental results reveals that there exists a relatively high (above 1%) disagreement in viscosity values obtained by individual authors, although there is good agreement among relative viscosity values in a number of recently performed studies. Thus, in developing the equation for viscosity as a function of temperature the compatible results of [12-14, 18-20] were chosen as the basic data. In the authors' opinion these results have the greatest accuracy with respect to relative values $-\mu^{rel}$. For the quantity μ_{20} the value utilized by each of the individual authors was employed.

TABLE 1. Experimental Studies of Water Viscosity at Atmospheric Pressure

Method	Temperature	Accuracy	Reference
Capillary absolute	0—45		1,2
«	0—50		3
«	0—100		4
«	0—100		5
«	0—100		6
«	0—97		7
Capillary rel.	—9,3—0		8
«	0—100		9
oscillatory cyl.	1—44		10
Capillary rel.	25—80	0,1	11
«	25—40		12
«	5—125	0,25	13
«	0—40	0,03—0,1	14
«	—25—0		15
Capillary absolute	12—27		16
Capillary rel.	0,22—150	0,6	17
«	25—150	0,09	18
«	8—70	0,05	19
«	—8,28—40		20

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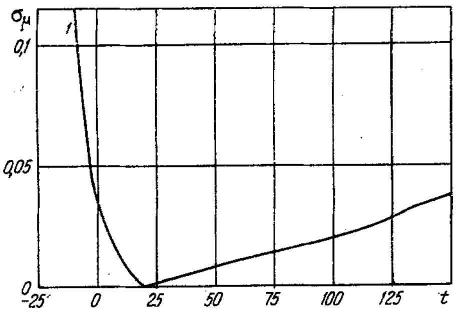

Fig. 1. Error in calculated values of μ^{rel}, σ_μ, rel. %; t, °C.

Absolute values of water viscosity at 20°C have been determined by a number of authors. A complete listing of such studies is given in [21], with the most reliable values, in our opinion, being those obtained by the following:

Swindells, Coe, Godfrey [22]	μ_{20}	= 1.0019 ± 0.0003,
Rosco, Bainbridge [23]	μ_{20}	= 1.0025 ± 0.0005,
Malyarov [21]	μ_{20}	= 1.0035 ± 0.0001.

The error shown here is the mean square value determined by the individual authors.

These three values of μ_{20} were also utilized in developing the equation.

To obtain the equation $\mu = f(T)$ an electronic digital computer program was developed which permitted simultaneous utilization of both absolute and relative viscosity values. All of the primary data for μ^{rel} referred to above were assigned a weight dependent on the error estimated by the authors. All μ_{20} values were given equal weight, since examination of [21] indicates that the accuracy of results was somewhat overestimated by the authors. The possibility of describing the temperature dependence of viscosity by various equations was also studied. Thus equations of the form

$$\ln \mu = \sum_{i=0}^{k} a_i \left(\frac{100}{T} \right)^i \tag{2}$$

and the form proposed in [20],

$$\mu^{rel} = \left(\frac{T}{273.15} \right)^n \exp \left[\frac{B(273.15 - T)}{(273.15 - C)(T - C)} \right], \tag{3}$$

were used, with the numerical values of the parameters a_i, n, B, C determined by the method of least squares.

Calculations revealed that Eq. (2) provides the closest approximation to the original experimental data. From the entire group of equations of this form, using the Fisher criterion at a reliability level of 0.95 [24], the final equation selected was

$$\ln \mu = \sum_{i=0}^{5} a_i \left(\frac{100}{T} \right)^i, \tag{4}$$

with the following coefficient values:

$a_0 = -1.1469663 \cdot 10^1$, $a_3 = 3.74665106 \cdot 10^3$,
$a_1 = 1.43659564 \cdot 10^2$, $a_4 = -7.02407628 \cdot 10^3$,
$a_2 = -9.97411315 \cdot 10^2$, $a_5 = 5.39493001 \cdot 10^3$.

TABLE 2. Dynamic Viscosity of Water, μ

T, °C	0	1	2	3	4	5	6	7	8	9
—20	4,242	4,022	3,819	3,630	3,455	3,293	3,141	3,000	2,868	2,745
—10	2,629	2,521	2,419	2,324	2,234	2,149	2,070	1,994	1,923	1,856
0	1,793	1,732	1,675	1,621	1,569	1,520	1,474	1,429	1,387	1,346
10	1,308	1,271	1,236	1,202	1,170	1,139	1,110	1,081	1,054	1,028
20	1,0026	0,9785	0,9553	0,9330	0,9115	0,8907	0,8708	0,8515	0,8330	0,8150
30	0,7978	0,7810	0,7649	0,7493	0,7342	0,7196	0,7054	0,6918	0,6785	0,6656
40	0,6532	0,6411	0,6294	0,6180	0,6070	0,5963	0,5859	0,5758	0,5660	0,5564
50	0,5471	0,5381	0,5293	0,5208	0,5124	0,5043	0,4964	0,4887	0,4812	0,4740
60	0,4668	0,4599	0,4531	0,4465	0,4401	0,4338	0,4276	0,4216	0,4158	0,4100
70	0,4045	0,3990	0,3937	0,3884	0,3834	0,3784	0,3735	0,3687	0,3640	0,3595
80	0,3550	0,3506	0,3463	0,3421	0,3380	0,3340	0,3300	0,3262	0,3224	0,3187
90	0,3150	0,3115	0,3080	0,3046	0,3012	0,2979	0,2946	0,2915	0,2884	0,2853
100	0,2823	0,2793	0,2764	0,2736	0,2708	0,2681	0,2654	0,2627	0,2601	0,2576
110	0,2551	0,2526	0,2502	0,2478	0,2454	0,2431	0,2409	0,2386	0,2364	0,2343
120	0,2322	0,2301	0,2280	0,2260	0,2240	0,2221	0,2201	0,2182	0,2164	0,2145
130	0,2127	0,2109	0,2092	0,2074	0,2058	0,2041	0,2024	0,2008	0,1992	0,1976
140	0,1960	0,1945	0,1930	0,1915	0,1900	0,1886	0,1871	0,1857	0,1843	0,1829
150	0,1816									

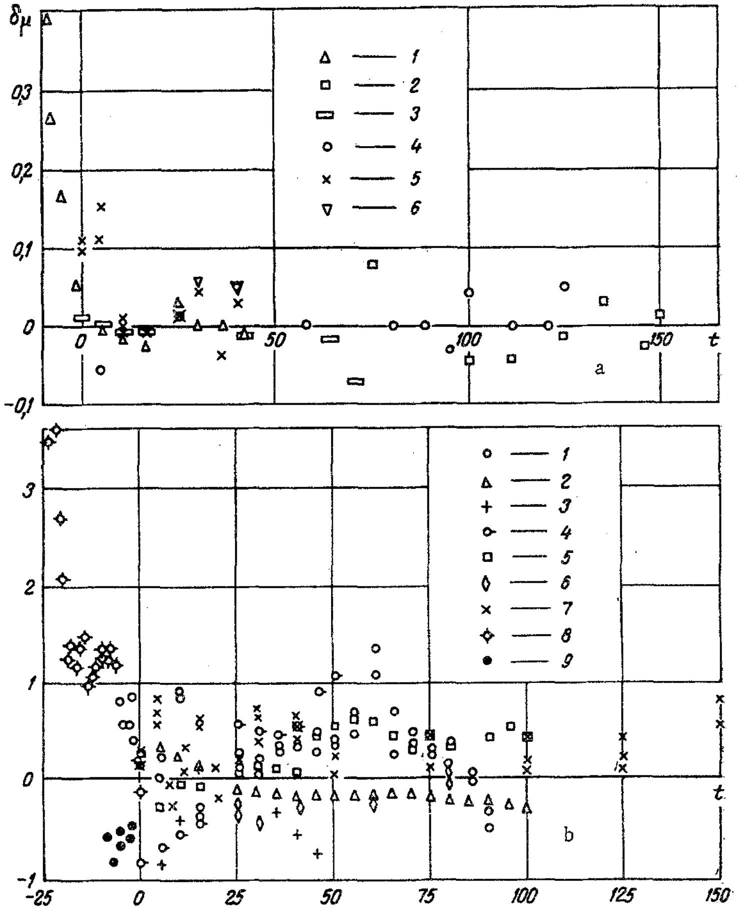

Fig. 2. Comparison of calculated μ^{rel} values with original experimental data: 1) [20]; 2) [18]; 3) [19]; 4) [13]; 5) [14]; 6) [12]; a1) [7]; 2) [9]; 3) [1]; 4) [3]; 5) [4]; 6) [11]; 7) [17]; 8) [15]; 9) [8]; b) δ_μ, rel. %, t, °C.

Values of dynamic viscosity of water at temperatures from −20 to 150°C, calculated in [4], are presented in Table 2. At temperatures below 0°C they refer to supercooled water, and above 100°C to water under saturation pressure.

It should be noted that Eq. (4) produces a viscosity value at 20°C of μ_{20} = 1.0026, which differs from the value often used at present, μ_{20} = 1.002. However it should be noted that the latter value is based on the data of one study [22] and may be corrected by consideration of later studies which give somewhat higher values of μ_{20}. Aside from the data mentioned above [21, 23] it may also be noted that Rivkin and Levin [25] also obtained a higher value μ_{20} = 1.0026. In [26] an indirect evaluation of the value of μ_{20} = 1.0026 was made by use of data on absolute values of viscosity of organic liquids obtained by various methods. The values of μ_{20} for water obtained in [26] lie within the limits 1.001-1.006, which in our opinion also indicates that μ_{20} is probably higher than 1.002. Performance of an exact statistical calculation of the error of the μ_{20} value obtained was not possible at present. However, analysis of a number of studies

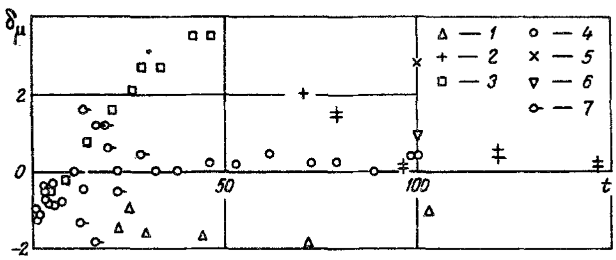

Fig. 3. Comparison of calculated viscosity values μ with experimental data: 1) [27]; 2) [29]; 3) [10]; 4) [5]; 5) [28]; 6) [30]; 7) [16] δ_μ, %; t, °C.

TABLE 3

T, °C	0	1	2	3	4	5	6	7	8	9
0	1,793	1,732	1,675	1,621	1,569	1,520	1,474	1,429	1,387	1,347
10	1,308	1,272	1,236	1,203	1,171	1,140	1,111	1,082	1,055	1,029
20	1,0044	0,9805	0,9574	0,9353	0,9139	0,8934	0,8736	0,8545	0,8361	0,8184
30	0,8012	0,7847	0,7687	0,7533	0,7383	0,7239	0,7099	0,6964	0,6833	0,6706
40	0,6583	0,6464	0,6348	0,6236	0,6127	0,6022	0,5919	0,5820	0,5723	0,5629
50	0,5538	0,5449	0,5362	0,5278	0,5196	0,5116	0,5039	0,4963	0,4890	0,4818
60	0,4748	0,4680	0,4614	0,4549	0,4485	0,4423	0,4364	0,4305	0,4247	0,4191
70	0,4137	0,4083	0,4031	0,3980	0,3930	0,3881	0,3834	0,3787	0,3741	0,3697
80	0,3653	0,3610	0,3568	0,3528	0,3487	0,3448	0,3410	0,3372	0,3335	0,3299
90	0,3264	0,3229	0,3195	0,3162	0,3129	0,3097	0,3065	0,3034	0,3004	0,2975
100	0,2945									

using different methods indicates an uncertainty $\sigma_{\mu_{20}} = 1 \cdot 10^{-3}$. Should a different value be taken for μ_{20}, the new value of μ may be obtained easily from Eq. (4), rewriting it in the form

$$\ln(\mu/\mu_{20}) = -11.472296 + \sum_{i=1}^{5} a_i \left(\frac{100}{T}\right)^i . \tag{5}$$

The error in relative viscosity values $\sigma_{\mu\,\mathrm{rel}}$, calculated with a covariation matrix obtained in developing Eq. (4) is shown in Fig. 1. As is evident from the figure, for the temperature range 0-100°C it is significantly lower than the estimated error in the absolute viscosity at 20°C $\sigma_{\mu_{20}}$, and thus the error in absolute viscosity values over this range is determined essentially by the value $\sigma_{\mu_{20}}$.

In Fig. 2a, a comparison of relative viscosity values calculated with Eq. (5) with the original experimental data is shown. A comparison of the calculated values with data from the literature not considered in formulating the equation is made in Figs. 2b, 3. For comparison, viscosity values obtained at pressures higher than atmospheric (up to 50 bar) or saturation pressure [27-30] are employed. Viscosity values were corrected to atmospheric pressure with an equation employed in compiling international water viscosity tables [31]. Because of the slight dependence of viscosity on pressure in this range of pressures and temperatures the correction cannot introduce significant error.

As is evident from Fig. 2b, despite the fact that in developing Eq. (4) data at temperatures below t = −8.28°C were not used, the viscosity values calculated with the equation at lower temperatures agree with the experimental data of [7, 15] within the limits of their accuracy.

Table 3 presents values of kinematic viscosity for water, calculated with the equation

$$\nu = \mu/\rho, \tag{6}$$

in which the water density value ρ was calculated from [32]. Since error in the density values is negligibly small in comparison to error in the dynamic viscosity values, the error in the kinematic viscosity values is completely determined by the latter.

NOTATION

μ, dynamic viscosity at temperature t, mp · sec; μ_{20}, dynamic viscosity at 20°C; μ^{rel}, relative viscosity; a_i, coefficients of Eq. (2); a, n, B, C, coefficients of Eq. (3); T, absolute temperature, °K; ν, kinematic viscosity, m^2/sec; σ, mean square error.

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